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Attempt to Resolve the EPR-Bell Paradox via Reichenbach's Concept of Common Cause†

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Reichenbach's common cause principle claims that if there is correlation between two events and none of them is directly causally influenced by the other, then there must exist a third event that can, as a common cause, account for the correlation. The EPR-Bell paradox consists in the problem that we observe correlations between spatially separated events in the EPR experiments which do not admit common-cause-type explanation, and it must therefore be concluded that, contrary to relativity theory, in the realm of quantum physics there exists action at a distance, or at least superluminal causal propagation is possible; that is, either relativity theory or Reichenbach's common cause principle fails. By means of closer analyses of the concept of common cause and a more precise reformulation of the EPR experimental scenario, I sharpen the conclusion we can draw from the violation of Bell's inequalities.

1. REICHENBACH'S DEFINITION OF COMMON CAUSE

Two events are correlated if the following holds for their probabilities:

$$\Delta_p(AB) = p(AB) - p(A)p(B) \neq 0$$

Seeing correlation between two events A and B, one can imagine two kinds of explanation: (1) the occurrence of one event is directly influenced by the occurrence of the other (so called *direct correlation*), or (2) the correlation is explained by the existence of a third event Z, a *common cause*, which is directly correlated to both A and B. In this case we say that the correlation is a *common cause correlation*.

Following Reichenbach [1], we give the following definition for a common cause:

† This paper is dedicated to the memory of Prof. Gottfried T. Rüttimann.

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Let A and B be two correlated events, $\Delta_p(AB) \neq 0$. An event Z is called common cause if

$$Z \neq A,B$$

$$\text{if} \quad Z \subset A \quad \text{or} \quad Z \supset A, \quad \text{then} \quad p(Z) \neq p(A) \tag{1}$$

$$\text{if} \quad Z \subset B \quad \text{or} \quad Z \supset B, \quad \text{then} \quad p(Z) \neq p(B) \tag{2}$$

$$p(AB|Z) = p(A|Z)p(B|Z) \tag{3}$$

$$p(AB|\overline{Z}) = p(A|\overline{Z})p(B|\overline{Z}) \tag{4}$$

$$\begin{aligned} &\begin{Bmatrix} p(AZ) > p(A)p(Z) \\ p(BZ) > p(B)p(Z) \end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix} p(AZ) < p(A)p(Z) \\ p(BZ) < p(B)p(Z) \end{Bmatrix} \quad \text{if} \quad \Delta(AB) > 0 \\ &\begin{Bmatrix} p(AZ) > p(A)p(Z) \\ p(BZ) < p(B)p(Z) \end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix} p(AZ) < p(A)p(Z) \\ p(BZ) > p(B)p(Z) \end{Bmatrix} \quad \text{if} \quad \Delta(AB) < 0 \end{aligned} \tag{5}$$

Only the so-called "screening off" properties (3) and (4) need some explanation: If we restrict the statistical ensemble to the subensemble in which the occurrence (or nonoccurrence, respectively) of the common cause event is fixed, then the correlation disappears.

2. COMMON CAUSE PRINCIPLE

Reichenbach's Common Cause Principle (CCP). If two events are correlated and one can exclude the possibility of direct causal relationship between them, then there must exist an event satisfying all the conditions required in the above definition of common cause.

In general, the original event algebra does not contain a common cause. This is not required by the CCP, but it requires the existence of a common cause event *in reality*. However, if for each correlation there is a common cause in reality, then we may with good reason assume that the original event algebra is extendable in such a way that all of these common causes are contained in the extension. Otherwise, we would find ourselves in an extremely counterintuitive situation as observing events in the world, about which we would not be able to speak with the logical connectives of everyday language.

It would be an obvious strategy to prove the invalidity of the CCP if someone showed correlated events described by a probabilistic model which were not extendable with common causes for all correlations. *This is typically the strategy of the EPR-Bell theorem aimed to prove the failure of the CCP in quantum mechanics*.

3. EPR-ASPECT EXPERIMENT

The four detectors in Fig. 1 detect the *spin-up* events in the spin-component measurements in directions **a**, **a**′ and **b**, **b**′. There are random switches (or independent agents) choosing between the different possible measurements on both sides. Let $p(a)$, $p(a')$ and $p(b)$, $p(b')$ be *arbitrary* probabilities with which the different measurements are chosen. We can experience the following events in the experiment:

A: "the spin of the left particle is *up* in direction **a**" detector fires

A': "the spin of the left particle is *up* in direction **a**′" detector fires

B: "the spin of the left particle is *up* in direction **b**" detector fires

B': "the spin of the left particle is *up* in direction **a**′" detector fires

a: the left switch chooses the **a**-measurement

a': the left switch chooses the **a**′ measurement

b: the right switch chooses the **b**-measurement

b': the right switch chooses the **b**′-measurement

Let **a**, **a**′, **b**, **b**′ be coplanar vectors such that $\sphericalangle(\mathbf{a}, \mathbf{a}') = \sphericalangle(\mathbf{a}', \mathbf{b}') = \sphericalangle(\mathbf{a}, \mathbf{b}') = 120^\circ$ and $\sphericalangle(\mathbf{a}', \mathbf{b}) = 0$. We observe the following relative frequencies in the experiment:

$$\begin{aligned} p(A) &= \tfrac{1}{2}p(a), & p(B) &= \tfrac{1}{2}p(b) \\ p(A') &= \tfrac{1}{2}p(a'), & p(B') &= \tfrac{1}{2}p(b') \\ p(AB) &= \tfrac{3}{8}p(a)p(b), & p(A'B) &= 0 \\ p(AB') &= \tfrac{3}{8}p(a)p(b'), & p(A'B') &= \tfrac{3}{8}p(a')p(b') \end{aligned} \tag{6}$$

There are correlations among the outcomes of the measurements performed on the left and the right particles:

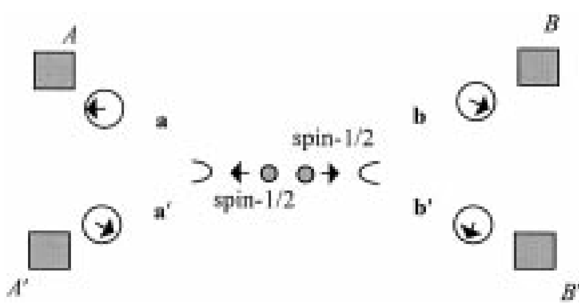

Fig. 1. The Aspect experiment with spin-1/2 particles.

$$\Delta_p(AB),\ \Delta_p(AB'),\ \Delta_p(A'B),\ \Delta_p(A'B') \neq 0 \tag{7}$$

The *usual interpretation* of the experimental data in (6) is the following: The conditional probability $p(A|a) \stackrel{\text{def}}{=} p(Aa)/p(a)$, for example, is regarded as the *quantum mechanical probability* $\mathrm{tr}(\hat{W}\hat{A})$ of a particular *quantum event* represented by projector $\hat{A}$. ($\hat{W}$ denotes the state operator of the system.) In this sense, the data in (6) are in accordance with the quantum mechanical prediction. Let us denote these probabilities as follows:

$$\begin{aligned}
q(A) &= p(A|a) = \tfrac{1}{2}, & q(AB) &= p(AB|ab) = \tfrac{3}{8} \\
q(A') &= p(A'|a') = \tfrac{1}{2}, & q(A'B) &= p(A'B|a'b) = 0 \\
q(B) &= p(B|b) = \tfrac{1}{2}, & q(AB') &= p(AB'|ab') = \tfrac{3}{8} \\
q(B') &= p(B'|b') = \tfrac{1}{2}, & q(A'B') &= p(A'B'|a'b') = \tfrac{3}{8}
\end{aligned} \tag{8}$$

The correlations in (7) can be expressed by using these quantum probabilities, too:

$$\begin{aligned}
\Delta_q(AB) = \Delta_q(AB') = \Delta_q(A'B') &= \tfrac{1}{8} \\
\Delta_q(A'B) &= -\tfrac{1}{4}
\end{aligned} \tag{9}$$

The question, as it is formulated everywhere in the literature of the EPR-Bell paradox, is this:

- Does a common cause for the correlations in (9) exist?

And the standard answer is: no!

4. WHY IS THIS QUESTION WRONG?

1. Small Mistake. It is tacitly assumed that all correlations encountered in the EPR experiment have the same common cause explanation. From the nonexistence of common common cause, however, it does not follow that CCP fails! One can easily show classical physical examples with correlations for which there is no common common cause, but which admit separate common causes.

2. Bad mistake. There are no events—and in principle, there cannot exist events in reality, the *relative frequencies* of which would be equal to "quantum probabilities" $q(A)$, $q(A')$, $q(B)$, ... In other words, the question we asked is a question about the existence of common cause for correlations among **nonexisting** events.

Argument. I don't know what a "quantum event" is, the probability of which is a number like

$$q(A) = \mathrm{tr}(\hat{W}\hat{A})$$

but anyone who knows should be able to tell a laboratory assistant when such an "event" occurs. According to the instruction the assistant makes a record like that in Table I.

Now, the relative frequencies are

$$\mathrm{v}(A) = \frac{N_A}{100000}, \ \mathrm{v}(A') = \frac{N_A}{100000},$$
$$\mathrm{v}(B) = \frac{N_S}{100000}, \ldots$$

In other words, the relative frequencies read off from the record are weighted averages of the classical truth-values: $\vec{\mathrm{v}} = \sum_{\varepsilon \in (0,1)^4} \lambda_\varepsilon \vec{u}^\varepsilon$, i.e., $\vec{\mathrm{v}} \in C(4, S)$. Consequently, numbers $\mathrm{v}(A)$, $\mathrm{v}(A')$, $\mathrm{v}(B)$, $\mathrm{v}(B')$, $\mathrm{v}(AB)$, $\mathrm{v}(AB')$, $\mathrm{v}(A'B)$, $\mathrm{v}(A'B')$ must satisfy the Bell–Clauser–Horne inequalities.

But $q(A)$, $q(A')$, $q(B)$, $q(B')$, $q(AB)$, $q(AB')$, $q(A'B)$, $q(A'B')$ do not satisfy the Bell–Clauser–Horne inequalities! (We used here Pitowsky's formalism [2], a brief account of which is given in the Appendix.) So, what the violation of the bell inequalities indicates is not that correlations (9) do not have common cause, but rather that there are no events which would have such correlations.

Table I

Run	A	A'	B	B'	AB	AB'	$A'B$	$A'B'$
1	√		√		√			
2	√							
3		√		√				√
4	√			√		√		
5			√					
6		√						
7		√		√				√
99995	√							
99996		√		√				√
99997	√			√		√		
99998			√					
99999		√						
100000		√		√				√
N	N_A	$N_{A'}$	N_B	$N_{B'}$	N_{AB}	$N_{AB'}$	$N_{A'B}$	$N_{A'B'}$

5. THE CORRECTED QUESTION

Stipulations

1. We restrict ourselves to the observed physical events A, A', B, B', a, a', b, b'.
2. For example, the left particle emitted from the source is subjected to one of the measurement procedures a or a'. The outcome of the measurement is affected by the preceding measurement procedure, and this effect shows regularities described by quantum mechanics.
3. Because of the spatial separation, the measurement outcome on one side must be independent of the measurement operation performed on the other side.
4. The choices of the measurements are free, therefore the left and right measurement selections are statistically independent.
5. We can speak about correlations between events on the left- and right-hand sides only in case of the outcomes of the measurements. Assume there is an event that is a common cause for such a correlation.
6. The probabilities of the measurement choices $p(a)$, $p(a')$, $p(b)$, $p(b')$, are entirely arbitrary. Consequently, one can require that the common cause event be a common cause independent of the concrete values of probabilities $p(a)$, $p(a')$, $p(b)$, $p(b')$.
7. Also, because the choices of the measurements are free in the sense that there is no mysterious conspiracy between the things that determines the choices of the measurements and those that determine the outcomes, one can assume that the measurement choices are independent of the common cause.
8. For sake of simplicity, we can finally assume that one of the measurements is surely performed on the both sides.

These findings are partly read off from the empirical data or they are straightforward consequences of the prohibition of superluminal causation.

The above requirements can be expressed in the following formulas:

New:

$$\begin{aligned} p(X) &= p(X|x)p(x) = \mathrm{tr}(\hat{W}\hat{X})p(x) \\ p(Y) &= p(Y|y)p(y) = \mathrm{tr}(\hat{W}\hat{Y})p(y) \end{aligned} \tag{10}$$

$$p(xy) = p(x)p(y) \tag{11}$$

$$\begin{aligned} p(Xy) &= p(X)p(y) \\ p(xY) &= p(x)p(Y) \end{aligned} \tag{12}$$

$$p(xZ_{XY}) = p(x)p(Z_{XY})$$
$$p(yZ_{XY}) = p(y)p(Z_{XY}) \tag{13}$$

$$p(a) + p(a') = p(b) + p(b') = 1 \tag{14}$$

$$Z_{XY} \neq X, Y \tag{15}$$

Reichenbach's:

$$\text{if} \quad Z \subset X \quad \text{or} \quad Z \supset X, \quad \text{then} \quad p(Z_{XY}) \neq p(X)$$
$$\text{if} \quad Z \subset Y \quad \text{or} \quad Z \supset Y, \quad \text{then} \quad p(Z_{XY}) \neq p(Y) \tag{16}$$

$$p(XY|Z_{XY}) = p(X|Z_{XY})\, p(Y|Z_{XY}) \tag{17}$$

$$p(XY|\overline{Z}_{XY}) = p(X|\overline{Z}_{XY})\, p(Y|\overline{Z}_{XY}) \tag{18}$$

$$\begin{Bmatrix} p(XZ_{XY}) > p(X)p(Z_{XY}) \\ p(YZ_{XY}) > p(Y)p(Z_{XY}) \end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix} p(XZ_{XY}) < p(X)p(Z_{XY}) \\ p(YZ_{XY}) < p(Y)p(Z_{XY}) \end{Bmatrix}$$
$$\text{if} \quad \Delta(XY) > 0 \tag{19}$$
$$\begin{Bmatrix} p(XZ_{XY}) > p(X)p(Z_{XY}) \\ p(YZ_{XY}) < p(Y)p(Z_{XY}) \end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix} p(XZ_{XY}) < p(X)p(Z_{XY}) \\ p(YZ_{XY}) > p(Y)p(Z_{XY}) \end{Bmatrix}$$
$$\text{if} \quad \Delta(XY) < 0$$

Here Z_{XY} denotes the common cause for correlation $\Delta(XY) \neq 0$, and $X = A$, A'; $Y = B$, B'; $x = a$, a'; $y = b$, b'.

Now, we correct the question about common cause in the following way:

- Do common causes for the (real) correlations in (7) exist satisfying conditions (10)–(19)?

6. ANSWERS

6.1. No Common Common Cause

If we also require that, in addition, the common causes for correlated pairs of events (A, B), (A, B'), (A', B), and (A', B') coincide,

$$Z_{AB} = Z_{AB'}Z_{A'B} = Z_{A'B'} = Z$$

then the answer is no. (For the Proof see http://xxx.lanl.gov/abs/quant-ph/9806074.)

6.2 There Exist Common Causes

The smallest algebra capable of representing the EPR events is shown in Fig. 2. The probabilities of the atoms of the algebra are

$$p_1 = p(a)p(b')(q(A) - q(AB'))$$
$$p_2 = p(a)p(b)(q(A) - q(AB))$$
$$p_3 = p(a)p(b')(1 - q(A) - q(B') + q(AB'))$$
$$p_4 = p(a)p(b)(1 - q(A) - q(B) + q(AB))$$
$$p_5 = p(a)p(b')q(AB')$$
$$p_6 = p(a)p(b)q(AB)$$
$$p_7 = p(a')p(b)(q(B) - q(A'B))$$
$$p_8 = p(a')p(b)(1 - q(A') - q(B) + q(A'B))$$
$$p_9 = p(a')p(b)(q(A') - q(A'B))$$
$$p_{10} = p(a')p(b')(q(A') - q(A'B'))$$
$$p_{11} = p(a')p(b')q(A'B')$$
$$p_{12} = p(a')p(b')(1 - q(A') - q(B') + q(A'B'))$$
$$p_{13} = p(a')p(b')(q(B') - q(A'B'))$$
$$p_{14} = p(a)p(b')(q(B') - q(AB'))$$
$$p_{14} = p(a)p(b)(q(B) - q(AB))$$

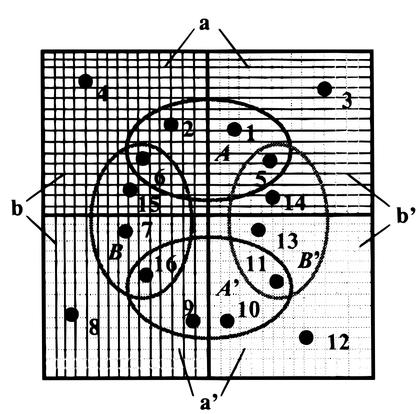

Fig. 2. The Boolean event algebra in which the events of the EPR-Aspect experiment can be represented.

$$p_{15} = p(a')p(b)q(A'B)$$

Consider now the extension shown in Fig. 3. The common cause events are represented by the following disjunction of blocks:

$$Z_{AB}:\quad 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$$

$$Z_{AB'}:\quad 1 + 2 + 5 + 6 + 9 + 10 + 13 + 14$$

$$Z_{A'B}:\quad 1 + 2 + 3 + 4 + 9 + 10 + 11 + 12$$

$$Z_{A'B'}:\quad 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$

The probabilities of the 256 atoms of the extended algebra can be chosen such that the four common cause events Z_{AB}, $Z_{AB'}$, $Z_{A'B}$, $Z_{A'B}$ satisfy all the required conditions (10)–(19). For the proof see http//:xxx.lanl.gov/abs/quant-ph/9806074.

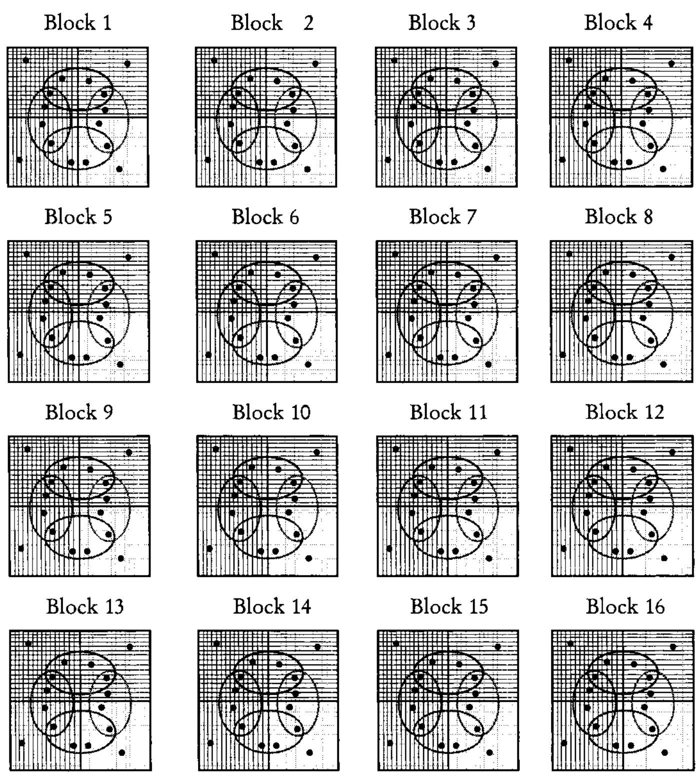

Fig. 3. The extension of the algebra shown in Fig. 2.

7. THE CHARGE OF "WORLD CONSPIRACY" AND RELATIVITY THEORY

The fact in itself that the measurement choices were taken into consideration (as events in the event algebra) does not necessarily lead to a "world conspiracy." The compatibility with relativity theory together with the "no world conspiracy" requirement means that:

- The left and right measurement choices are independent.
- The outcome on the one side is independent of the measurement operation on the other side.
- The common cause events are independent of the measurement operations
- Our model does satisfy all these conditions!

8. SHORTCOMINGS, OPEN QUESTIONS, CONCLUSIONS

- Our common cause model of the EPR-Aspect experiment satisfies all conditions required in the EPR-Bell literature. *In this sense it resolves the EPR-Bell paradox*.
- However, it is not a complete resolution of the paradox because there is a shortcoming of the model: While it is true that each common cause event is independent of the measurement choices, it turns out that such events as

$$Z_{AB} \wedge Z_{AB'}$$

$$Z_{AB} \vee Z_{AB'}$$

$$Z_{AB} \wedge Z_{AB'} \wedge Z_{A'B}$$

$$\text{etc.}$$

  may be not independent of the measurement operations. *It is still an open question whether there exists a modification of the model in which the above-mentioned events, too, are statistically independent of the measurement choices*.
- It is still an open question, of course, what kind of physical reality corresponds to these common cause models.

APPENDIX

Let S be a set of pairs of integers $S \subseteq \{\{i, j\} \mid 1 \leq i \leq j \leq n\}$. Denote by $R(n,S)$ the liner space of real vectors of form $(f_1, f_3, \ldots, f_n, \ldots f_y, \ldots)$. For each $\varepsilon \in \{0, 1\}^n$, let $\vec{u}^{\varepsilon}$ be the following vector in $R(n, S)$:

$$u_i^\varepsilon = \varepsilon_l,\ 1 \le i \le n$$
$$u_{ij}^\varepsilon = \varepsilon_i \varepsilon_j,\ \{i, j\} \in S \tag{A1}$$

The classical correlation polytope $C(n,S)$ is the closed convex hull in $R(n, S)$ of vectors $\{\vec{u}^\varepsilon\}_{z \in (0,1\text{cp}^n}$:

$$C(n, s) := \left\{ \vec{a} \in R(n, S) | \vec{a} = \sum_{z \in (0,1)^n} \lambda_z \vec{u}^z \text{ such that } \lambda_z \ge 0 \text{ and } \sum \lambda_z = 1 \right\} \tag{A2}$$

From the definition of the polytope, Equations (A1) and (A2), it follows that condition $\vec{p} \in C(n, S)$ equivalently means that the probabilities can be represented as weighted averages of the classical truth values.

In the case $n = 4$ and $S = \{\{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}\}$, the condition $\vec{p} \in C(n, S)$ is equivalent to the following Clauser–Horne inequalities:

$$0 \le p(A_i \wedge A_j) \le p(A_i) \le 1$$
$$0 \le p(A_i \wedge A_j) \le p(A_j) \le 1, \quad i = 1, 2, \quad j = 3, 4$$
$$p(A_i) + p(A_j) - p(A_i \wedge A_j) \le 1$$
$$-1 \le p(A_1 \wedge A_3) + p(A_1 \wedge A_4) + p(A_2 \wedge A_4) - p(A_2 \wedge A_3) - p(A_1) - p(A_4) \le 0$$
$$-1 \le p(A_2 \wedge A_3) + p(A_2 \wedge A_4) + p(A_1 \wedge A_4) - p(A_1 \wedge A_3) - p(A_2) - p(A_4) \le 0$$
$$-1 \le p(A_1 \wedge A_4) + p(A_1 \wedge A_3) + p(A_2 \wedge A_3) - p(A_2 \wedge A_4) - p(A_1) - p(A_3) \le 0$$
$$-1 \le p(A_2 \wedge A_4) + p(A_2 \wedge A_3) + p(A_1 \wedge A_3) - p(A_1 \wedge A_4) - p(A_2) - p(A_3) \le 0$$

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